Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

Tuesday 1 November 2022 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following table shows the Mathematics test scores (x) and the Science test scores (y) for a group of eight students.

Mathematics scores (x)	64	68	72	75	80	82	85	86
Science scores (y)	67	72	77	76	84	83	89	91

The regression line of y on x for this data can be written in the form $y = ax + b$.

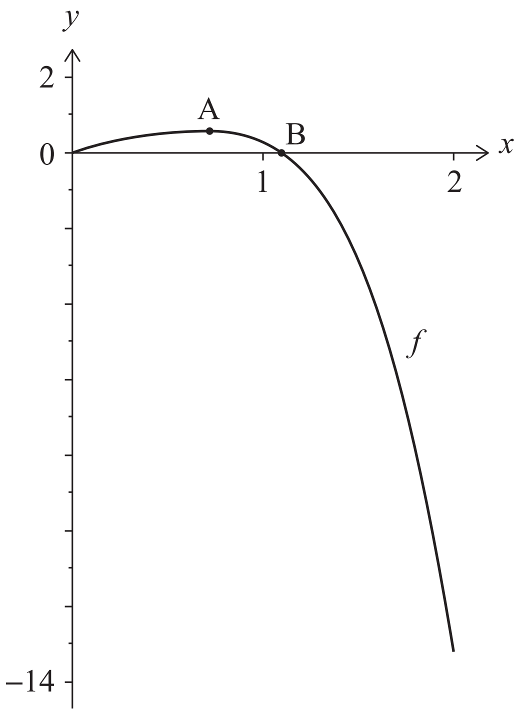
(a) Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use the equation of your regression line to predict the Science test score for a student who has a score of 78 on the Mathematics test. Express your answer to the nearest integer. [2]

2. [Maximum mark: 6]

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \le x \le 2$. The graph of f is shown in the following diagram.



The graph of f has a local maximum at point A. The graph intersects the x-axis at the origin and at point B.

(a) Find the coordinates of A. [2]

(b) Find the x-coordinate of B. [1]

(c) Find the total area enclosed by the graph of f, the x-axis and the line $x = 2$. [3]

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3. [Maximum mark: 5]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

4. [Maximum mark: 7]

The population of a town t years after 1 January 2014 can be modelled by the function

$$P(t) = 15\,000\mathrm{e}^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by 11%.

Use this model to estimate the population of this town on 1 January 2041.

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5. [Maximum mark: 6]

Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$.

Find the value of a.

6. [Maximum mark: 8]

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} axe^{x}, & 0 \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

where $a, b \in \mathbb{R}^{+}$.

(a) Find an expression for a in terms of b. [5]

(b) In the case where $a = b = 1$, find the median of X. [3]

7. [Maximum mark: 6]

Consider the vectors $\boldsymbol{u} = \boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{v} = \left(\cos\frac{1}{n}\right)\boldsymbol{i} + \left(\sin\frac{1}{n}\right)\boldsymbol{j}$, where $n \in \mathbb{Z}^{+}$.

Let θ be the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

(a) Find an expression for $\cos\theta$ in terms of n. [3]

(b) Find the exact value of the limit approached by θ as $n \to \infty$. [3]

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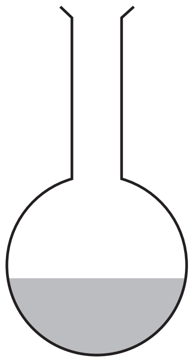
8. [Maximum mark: 6]

The following diagram shows liquid in a round-bottomed glass flask, which is made of a sphere and a cylindrical neck.



Initially, the flask is empty. Liquid is poured into the flask at a rate of $2\,\text{cm}^3\text{s}^{-1}$. You may assume that the liquid does not reach the cylindrical neck.

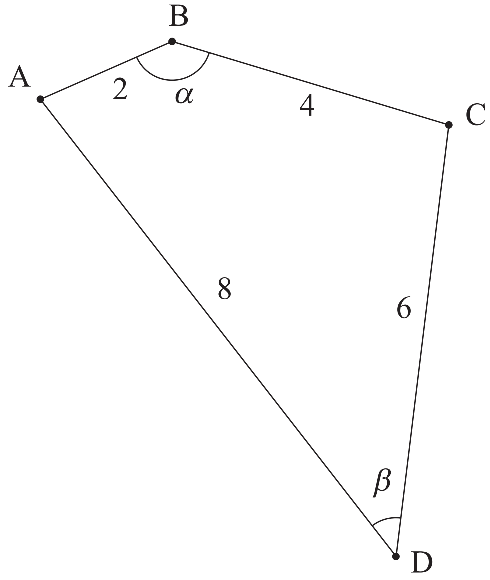
The volume $V\,\text{cm}^3$ and the height $h\,\text{cm}$ of the liquid in the flask satisfy the equation $V = 5\pi h^2 - \frac{1}{3}\pi h^3$.

Find the rate of change of the height of the liquid in the flask at the instant when the volume of the liquid is $200\,\text{cm}^3$.

9. [Maximum mark: 8]

Consider a quadrilateral ABCD such that $\mathrm{AB} = 2$, $\mathrm{BC} = 4$, $\mathrm{CD} = 6$ and $\mathrm{DA} = 8$, as shown in the following diagram. Let $\alpha = \mathrm{A\hat{B}C}$ and $\beta = \mathrm{A\hat{D}C}$.

diagram not to scale



(a) (i) Find AC in terms of α .

(ii) Find AC in terms of β .

(iii) Hence or otherwise, find an expression for α in terms of β. [4]

(b) Find the maximum area of the quadrilateral ABCD. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

The time worked, T, in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

(a) Find the probability that an employee selected at random works more than 40 hours per week. [2]

(b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week. [3]

(c) A large group of employees work more than 40 hours per week.

(i) An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week.

(ii) Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week. [7]

It is known that $\mathrm{P}(a \le T \le b) = 0.904$ and that $\mathrm{P}(T > b) = 2\mathrm{P}(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

(d) Find the maximum time, in hours per week, that an employee can work and still be considered part-time. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 15]

The function f is defined by $f(x) = e^{2x}(3x - 4)$, where $x \in \mathbb{R}$.

(a) Find $f'(x)$. [3]

(b) Hence or otherwise, find the coordinates of the point on the graph of $y = f(x)$ where the tangent is parallel to the line $y = x$. [3]

The region enclosed by the curve $y = f(x)$, the x-axis and the y-axis is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the volume of this solid. [4]

Consider a function g such that $g(0) = 1$ and $g'(0) = 2$.

(d) Find the value of

(i) $(f \circ g)(0)$;

(ii) $(f \circ g)'(0)$. [5]

12. [Maximum mark: 22]

Consider the points $A(1, 2, 3)$, $B(k, -2, 1)$ and $C(5, 0, 2)$, where $k \in \mathbb{R}$.

(a) Write down $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

(b) Given that the points A, B and C lie on a straight line, show that $k = 9$. [1]

(c) For $k = 9$, let L_1 be the line passing through A, B and C.

(i) Find a vector equation of the line L_1.

(ii) Line L_2 has the equation $\frac{x-1}{2} = \frac{y}{3} = 1 - z$. Show that the lines L_1 and L_2 are skew. [10]

(d) For $k \neq 9$, let Π be the plane containing A, B and C.

(i) Find the Cartesian equation of the plane Π.

(ii) Find the coordinates of the point on the plane Π which is closest to the origin $(0, 0, 0)$. [9]

References: